Introduction to Quantum Information Technologies

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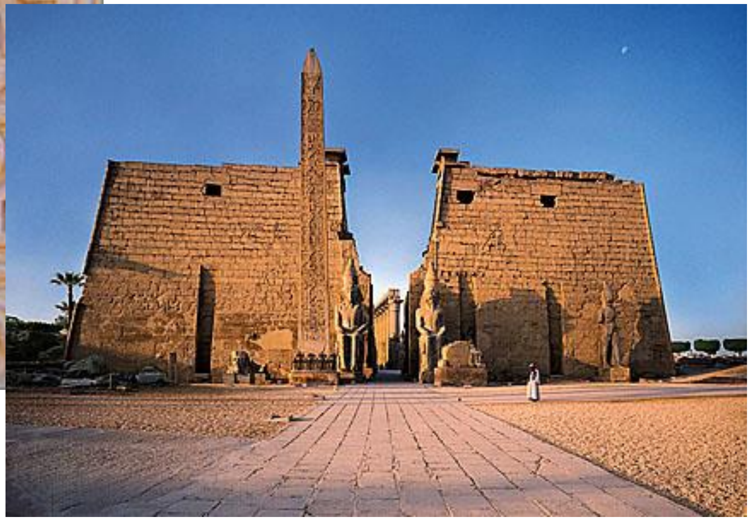
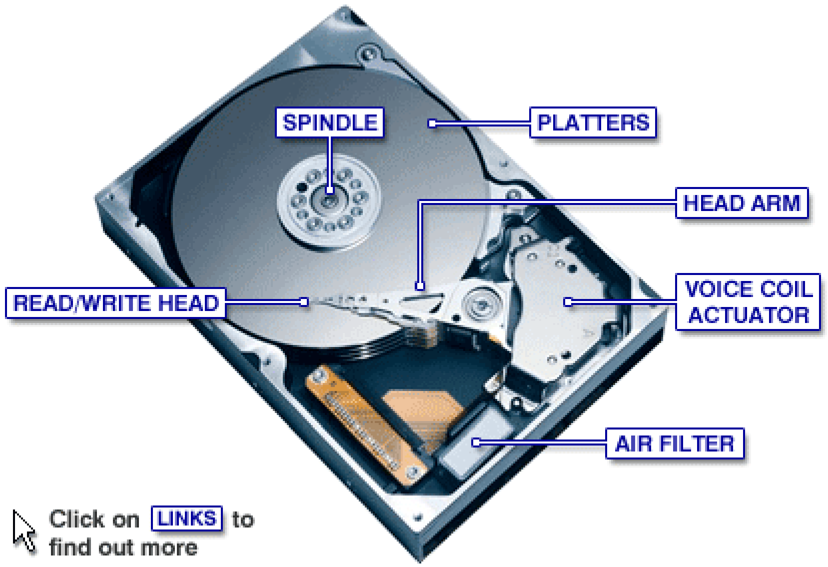
How long can we store a bit

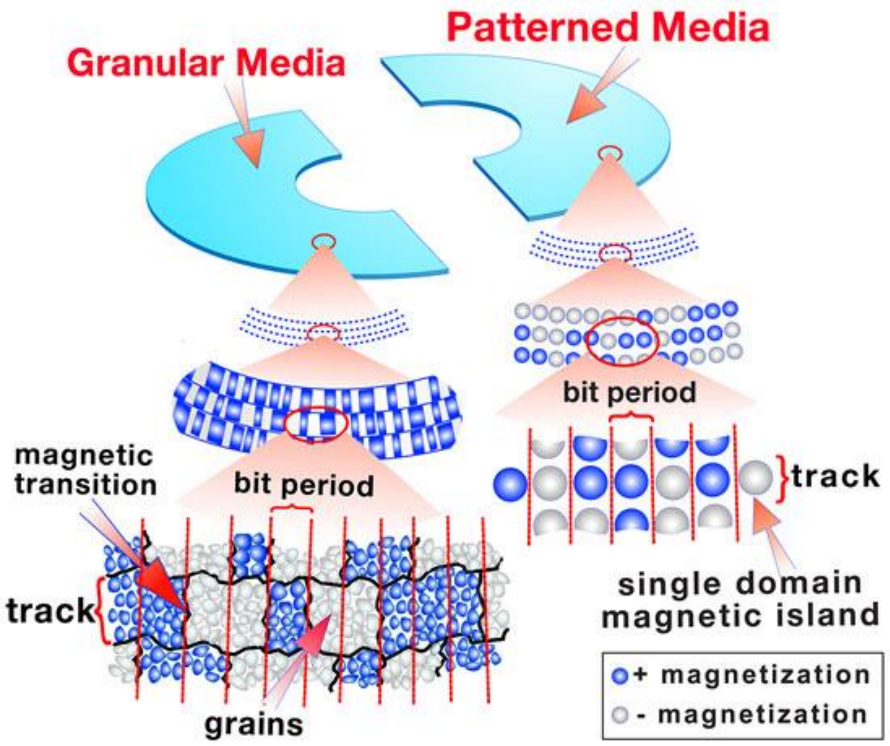


Hieroglyphs carved in sandstone at the Luxor Temple in Egypt (founded in 1400 BC).

How dense can we store bits?



Currently: almost 1 Terabit (10^{12}) bits per square inch.

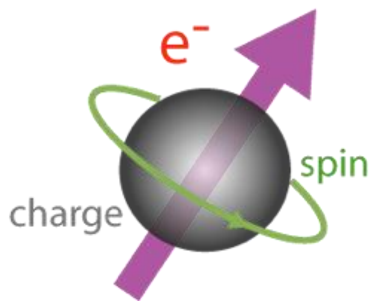


Bit is stored in magnetic grains.
Each individual grain has a fixed magnetization (positive + or negative -).
Transition from area with + grains to – grains represents the bit 1.
Photo: Hitachi Global Storage Technologies.

How do magnetic grains hold their magnetization?

Magnetization

Magnetic grains are made from a ferrimagnetic material and are ≥ 10 nanometer in size.
Ferromagnetism is explained by the mutual alignment of the electron spins of electrons bound at atoms in the material due to a so-called Heisenberg exchange interaction.
The electron spin is an example of a quantum bit, qubit.



At sufficiently low temperature $T < T_c$, excitations (spin waves) preserve the overall magnetization preference, at least, in the theoretical Heisenberg models in three dimensions, in the limit of a large number of electron spins.

Magnetic grains have finite size V (the smaller the better) and thermal effects can reverse magnetization, namely the probability for reversal scales as exp(-V/kT).
Superparamagnetic Limit.

What does this tell us..

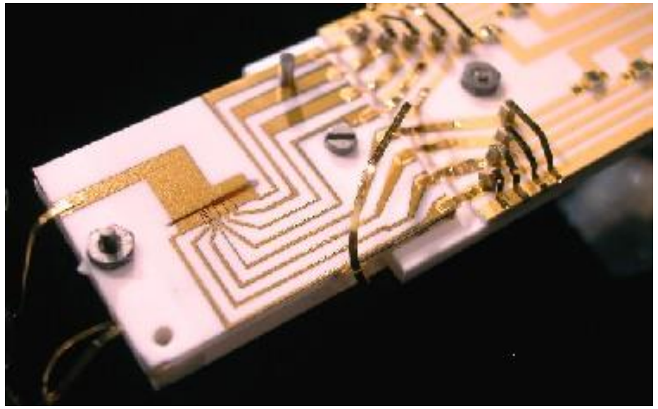
There are fundamental limitations to miniaturization due to the fact that we need to encode information redundantly (many electrons are needed, individual electron spins will fluctuate, but overall magnetization can be stable). This is a form of hard-ware based error correction.

We cannot just encode classical information in single spins, photons, atomic states, because these information carriers are only stable on very short time-scales.

But this is just what we are trying to do with quantum information….

Quantum information…

Last 15+ years have seen steady impressive progress in making and controlling single qubits…

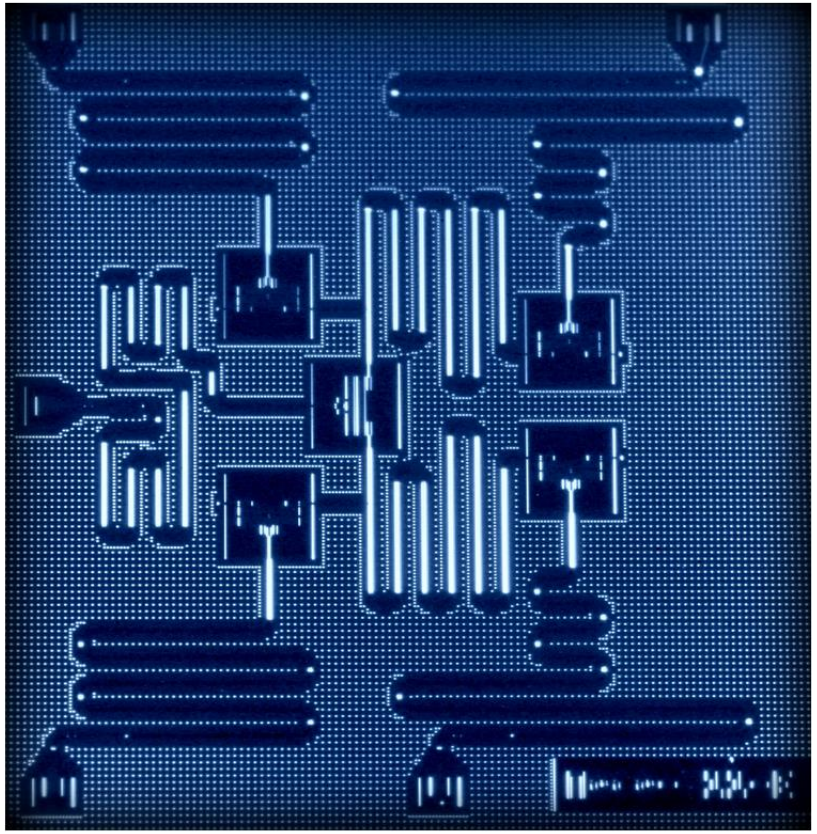


Trapped ion qubits (2009, NIST) Now have decoherence times of O(10) seconds.

Two superconducting transmon qubits (2009, Yale). Now decoherence times of 100 μseconds.

Decoherence times compares badly to those of Egyptian hieroglyphs…

IBM Quantum Experience chip



- Resonators/couplers O(1) mm in size
- `Qubits`: square boxes.

- Error rate CR (CNOT) gate is approx. 0.01-0.05
- Error rate single-qubit gate is approx. 0.0001.

http://www.research.ibm.com/quantum/

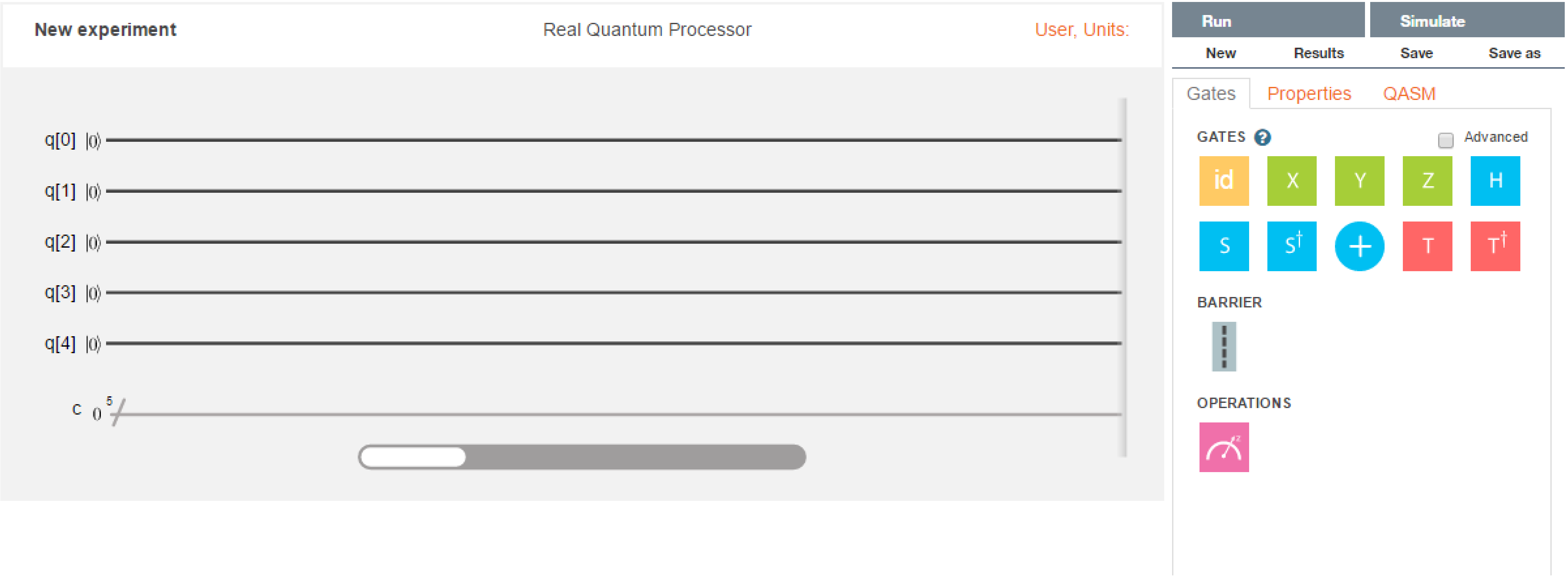
Specs on current online device in the cloud:

Single-qubit gate length: 83 ns

Two-qubit gate lengths: CR01 (416 ns), CR02 (367 ns), CR12 (483 ns), CR32 (416 ns), CR34 (350 ns), CR42 (350 ns)

Readout time: 2 μs

Interface


New experiment
Real Quantum Processor
User, Units:
Run
Simulate
New
Results
Save
Save as
Gates
Properties
QASM
GATES
Advanced
q[0] |0⟩
q[1] |0⟩
q[2] |0⟩
q[3] |0⟩
q[4] |0⟩
c 0 5
id
X
Y
Z
H
S
S†
T
T†
BARRIER
OPERATIONS

Gate Manual

Help

U1

The first physical gate of the Quantum Experience. It is a one parameter single-qubit phase gate with zero duration.

QASM | Matrix

U2

The second physical gate of the Quantum Experience. It is a two parameter single-qubit gate with duration one unit of time.

QASM | Matrix

U3

The third physical gate of the Quantum Experience. It is a three-parameter single-qubit gate with duration 2 units of gate time.

QASM | Matrix

id

The identity gate performs an idle operation on the qubit for a time equal to one unit of time.

QASM | Matrix

X

The Pauli X gate is a π-rotation around the X axis and has the property that $X \to X$, $Z \to -Z$. Also referred to as a bit-flip.

QASM | Matrix

Y

The Pauli Y gate is a π-rotation around the Y axis and has the property that $X \to -X$, $Z \to -Z$. This is both a bit-flip and a phase-flip, and satisfies $Y = XZ$.

QASM | Matrix

Z

The Pauli Z gate is a π-rotation around the Z axis and has the property that $X \to -X$, $Z \to Z$. Also referred to as a phase-flip.

QASM | Matrix

H

The Hadamard gate has the property that it maps $X \to Z$, and $Z \to X$. This gate is required to make superpositions.

QASM | Matrix

S

The Phase gate that is $\sqrt{Z}$ and has the property that it maps $X \to Y$ and $Z \to Z$. This gate extends H to make complex superpositions.

QASM | Matrix

$S^\dagger$

The Phase gate that is the transposed conjugate of S and has the property that it maps $X \to -Y$, and $Z \to Z$.

QASM | Matrix

$\oplus$

Controlled-NOT gate: a two-qubit gate that flips the target qubit (i.e. applies Pauli X) if the control is in state 1. This gate is required to generate entanglement and is the physical two qubit gate.

QASM | Matrix

T

The Phase gate that is $\sqrt{S}$, which is a $\pi/4$ rotation around the Z axis. This gate is required for universal control.

QASM | Matrix

$T^\dagger$

The Phase gate that is the transposed conjugate of T.

QASM | Matrix

The barrier prevents transformations across this source line.

QASM | Matrix

Measurement in the computational (standard) basis (Z).

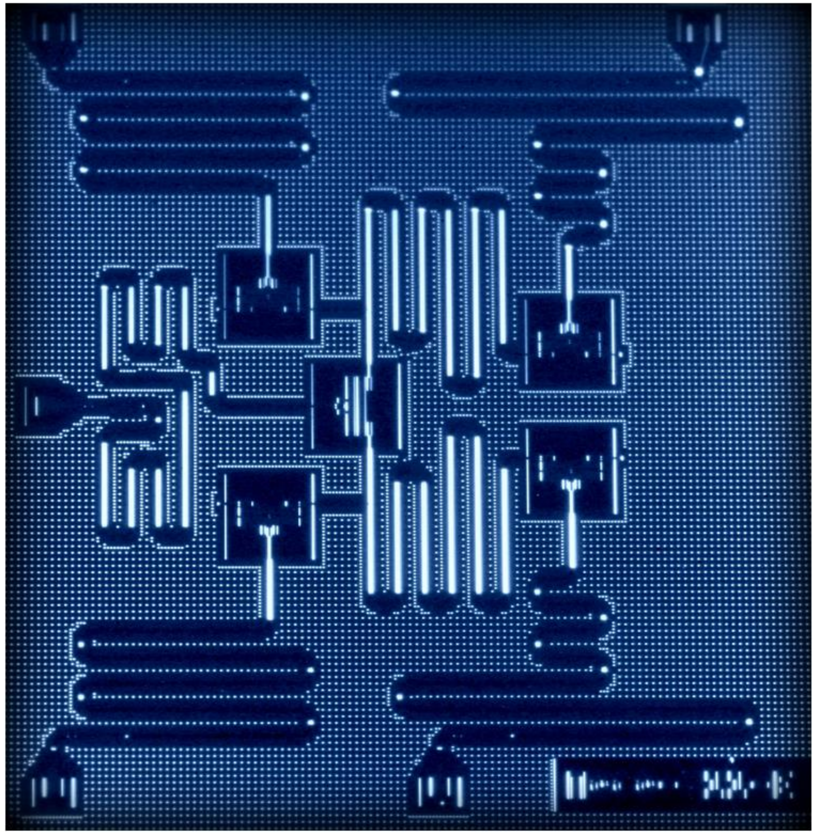
QASM | Matrix

if

Conditionally apply quantum operation

QASM | Matrix

IBM Quantum Experience chip



- Resonators/couplers O(1) mm in size
- Qubits: square boxes.

- Error rate CR gate is approx. 0.01-0.05
- Error rate single-qubit gate is approx. 0.0001.

After 100-1000 operations, game is over. Quantum error correction is an absolute necessity for building a computer.

Specs on current online device in the cloud:

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Readout time: 2 μs

http://www.research.ibm.com/quantum/

Qubits and their errors

A bit |0> or |1>, or bitstring |01110…> can undergo bitflip errors.

Represent 0 as $\begin{pmatrix}1\\0\end{pmatrix}$ and 1 as $\begin{pmatrix}0\\1\end{pmatrix}$.

Bitflip is Pauli X matrix: $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

A qubit $\alpha|0> +\beta|1>$, $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ can undergo bitflip (X) errors, or phaseflip (Z) errors (or both Y~X Z).

Phaseflip is

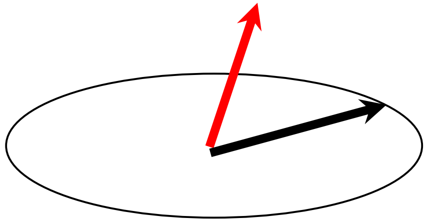
$$|0> \rightarrow |0>$$

$$|1> \rightarrow -|1>$$

represented by Pauli Z matrix: $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

The trouble with quantum information is that we have to correct or suppress both bit as well as phaseflip errors.

Quantum Error Correction

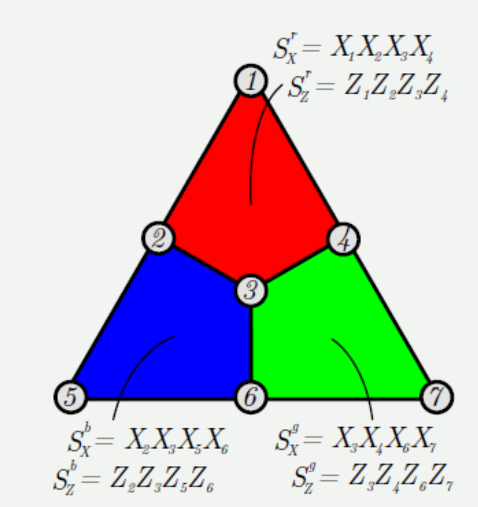


Early example: Steane’s 7 qubit code which encodes 1 qubit into 7 and corrects 1 error (distance 3). [[n=7,k=1,d=3]]

1. Popular family of quantum codes are stabilizer codes for which code space is defined as satisfying all parity checks.

Three-bit repetition code: 0 -> 000, 1 -> 111.
Parity checks Z_1Z_2, Z_1Z_3 have eigenvalue 1 on codewords (of the form $\alpha|000> + \beta|111>$). Bad quantum code: $Z_1(|000\rangle + |111\rangle) = |000\rangle - |111\rangle$.

Quantum codes also have parity checks which involve $X, i\,XZ = Y$, and parity check operators all mutually commute.



(2. How to do encoded gates?)

Quantum Error Correction

Error correction:

1. measure parity check operators.

2. Infer from measurements (error syndrome) what error most likely has occurred (by classical processing): decoder

Classical linear binary codes:

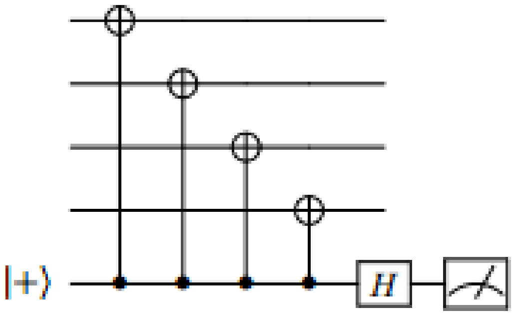
$H\ c = 0$ for codewords c and parity check matrix H.

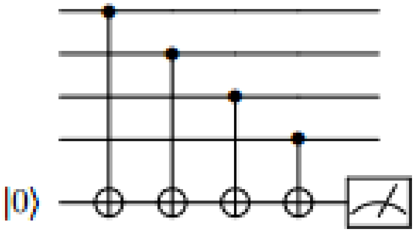
$H(c + e) = s$ with error e and syndrome s.

Parity check matrix of Hamming code (7,4) encoding 4 bits into 7.

$$\mathbf{H} := \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}_{3,7} .$$

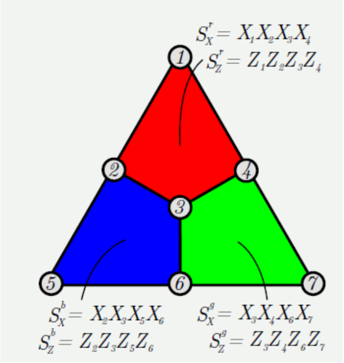
XXXX measurement





ZZZZ measurement

Parity check measurements



Steane code uses parity checks of Hamming code (7,4) for X and Z

Important parameters of codes

- Rate of code k/n=number of encoded qubits/number of physical qubits. High
- Distance d of code. Quantumly: minimum number of qubits on which a logical operator acts (mapping one logical state onto another). High
- Parity check weight & qubit degree= # 1s in rows and columns of parity check matrix: Low. LDPC code
- Noise threshold: high
- Efficient decoder.
- Parity checks local in 2D/3D. Bravyi-Terhal-Poulin bound in 2D: $kd^2 = O(n)$

Class of quantum codes associated with topology.

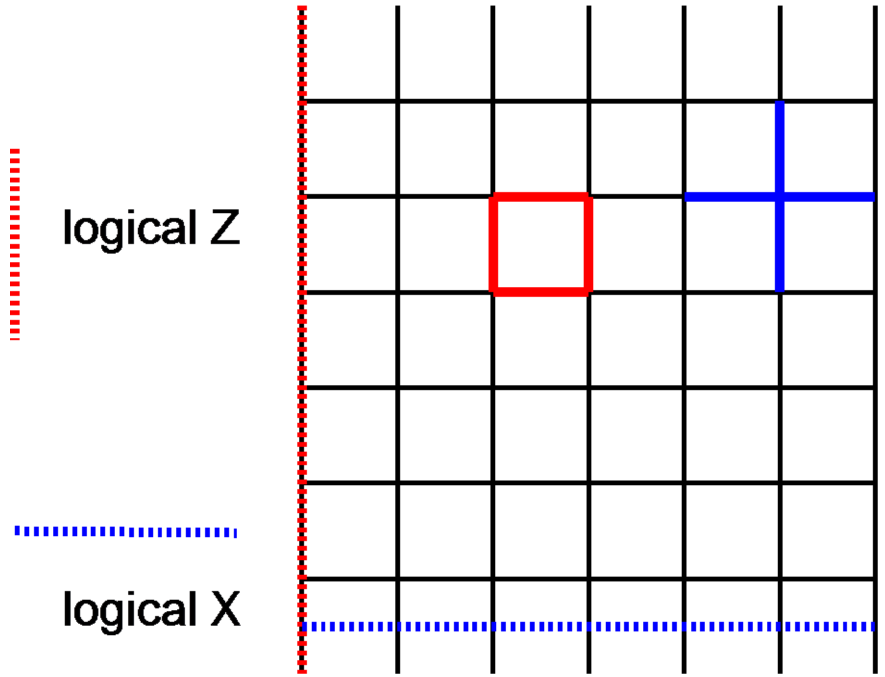
Surface code and hyperbolic surface codes.

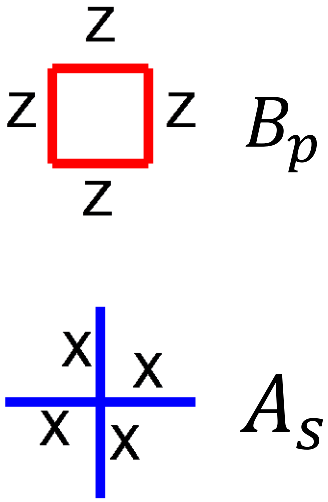
Surface Code

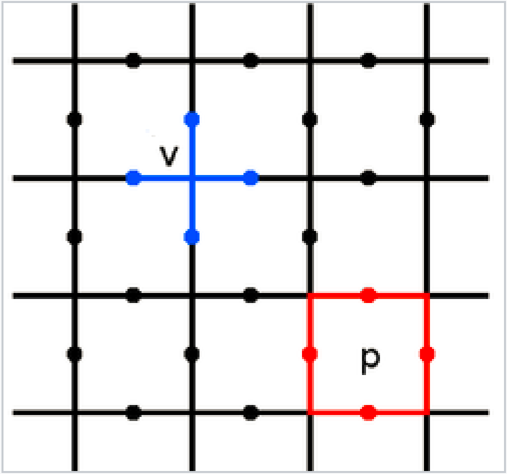
Surface code for storing 1 logical qubit $[[n = d^2 + (d-1)^2, k = 1, d]]$

Exponentially suppressed error rate on encoded qubit
$P_{enc} \sim (\frac{p}{p_c})^{d/2}$ where noise threshold $p_c \approx 0.6 - 0.75\%$

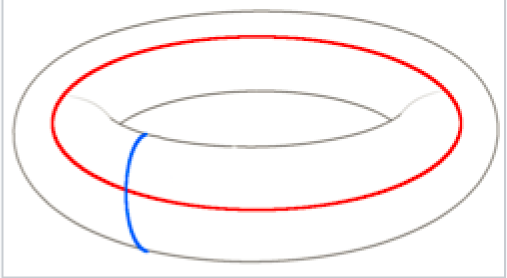
n qubits on edges.







A section of the toric code. A vertex and plaquette are highlighted, along with the spins used in the definition of their stabilizers.

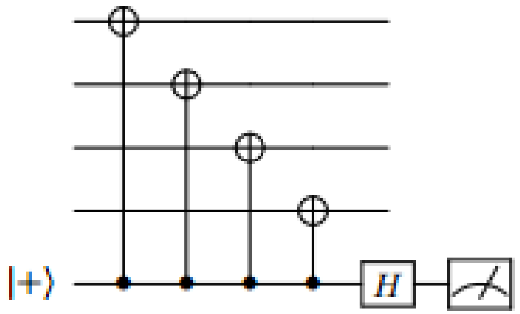


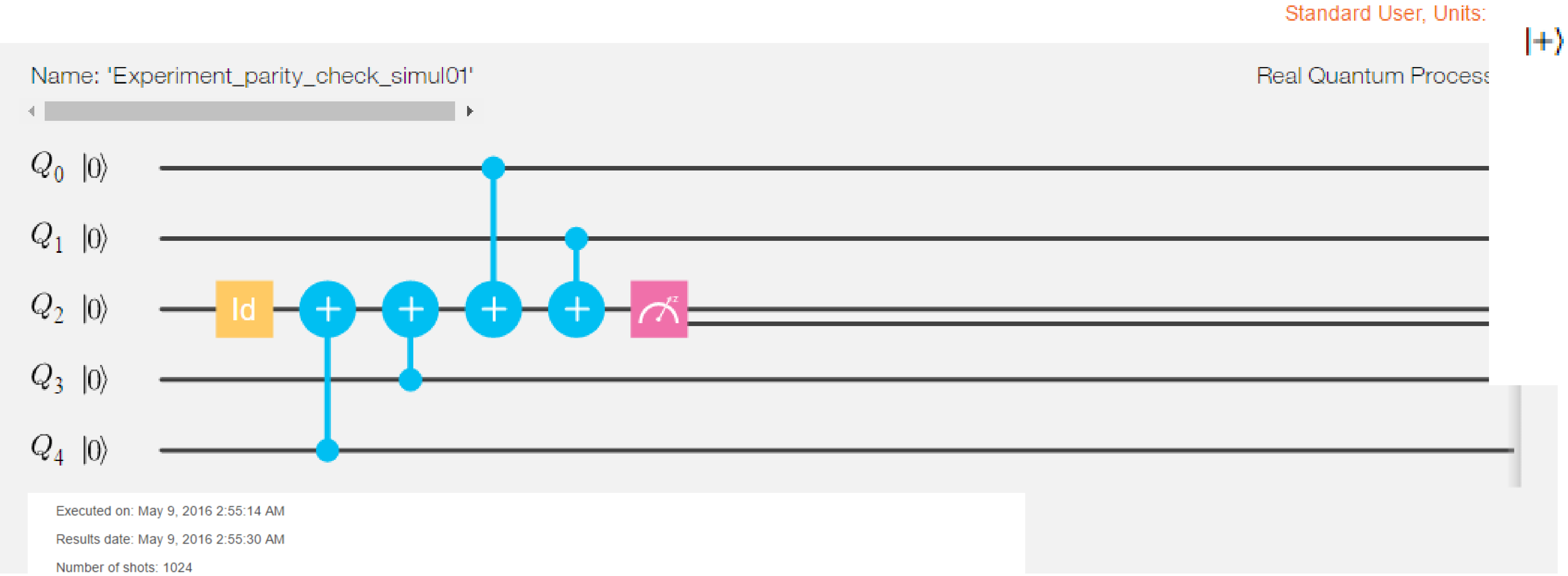
The topologically nontrivial loops of the torus. Moving anyons along these implement logical Pauli operators on the stored qubits.

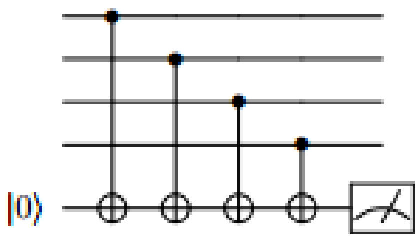
Toric code

Doing a parity check measurement on the IBM Quantum Experience

XXXX measurement





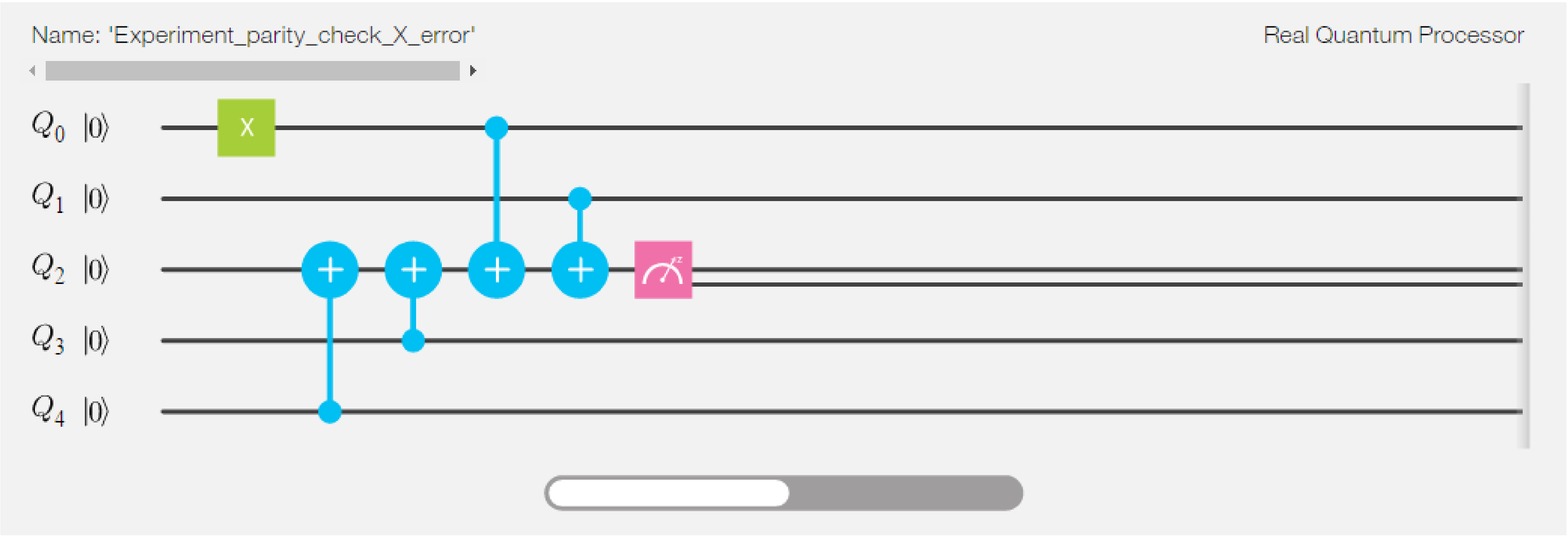


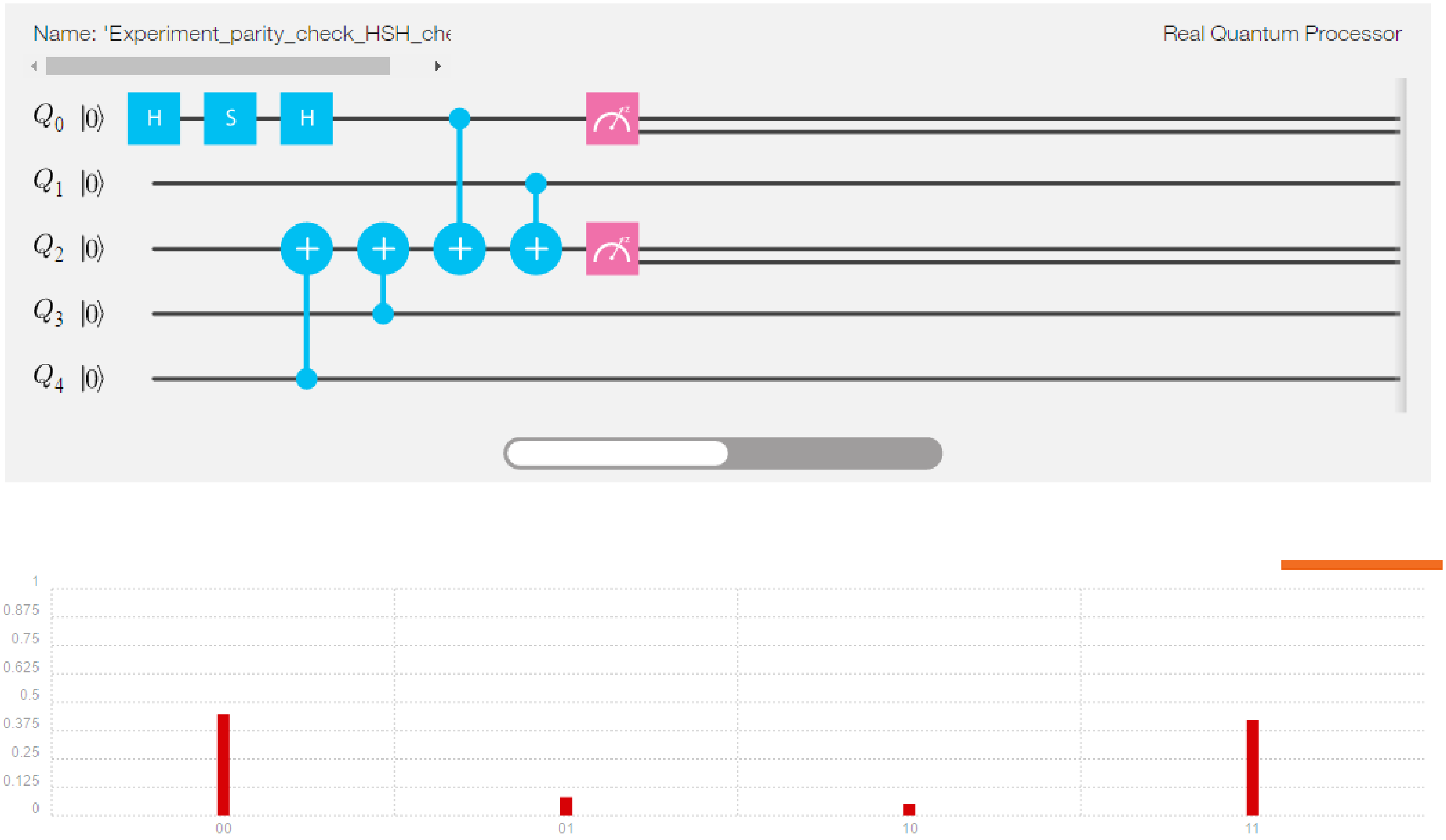
ZZZZ measurement

Doing a ZZZZ measurement here: is parity of bits even or odd.

Answer is wrong in 12.5% of the cases.

Standard User, Units: 0/15





Statistics of 4000 shots

Surface Code in Progress

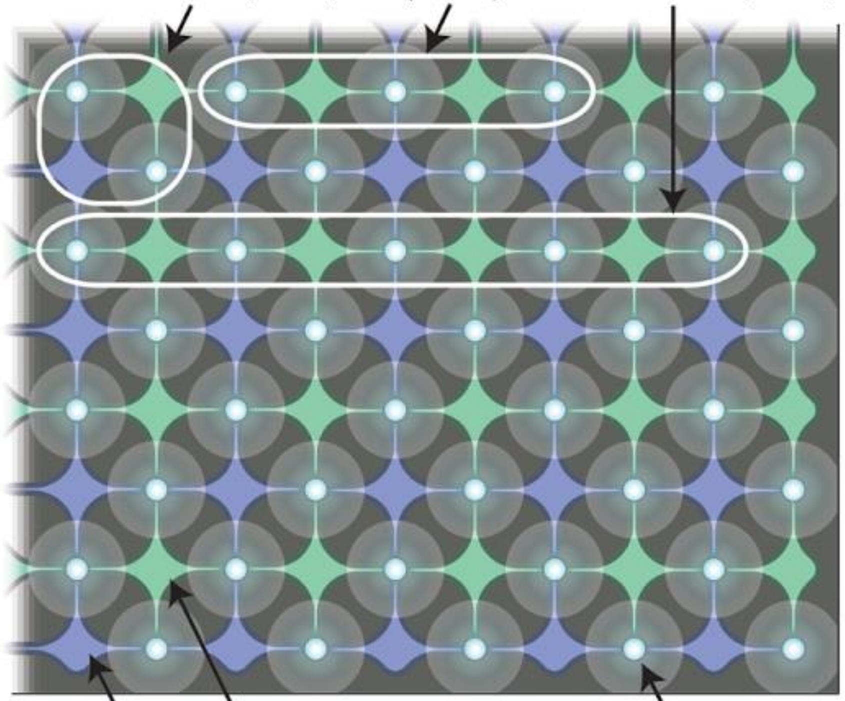
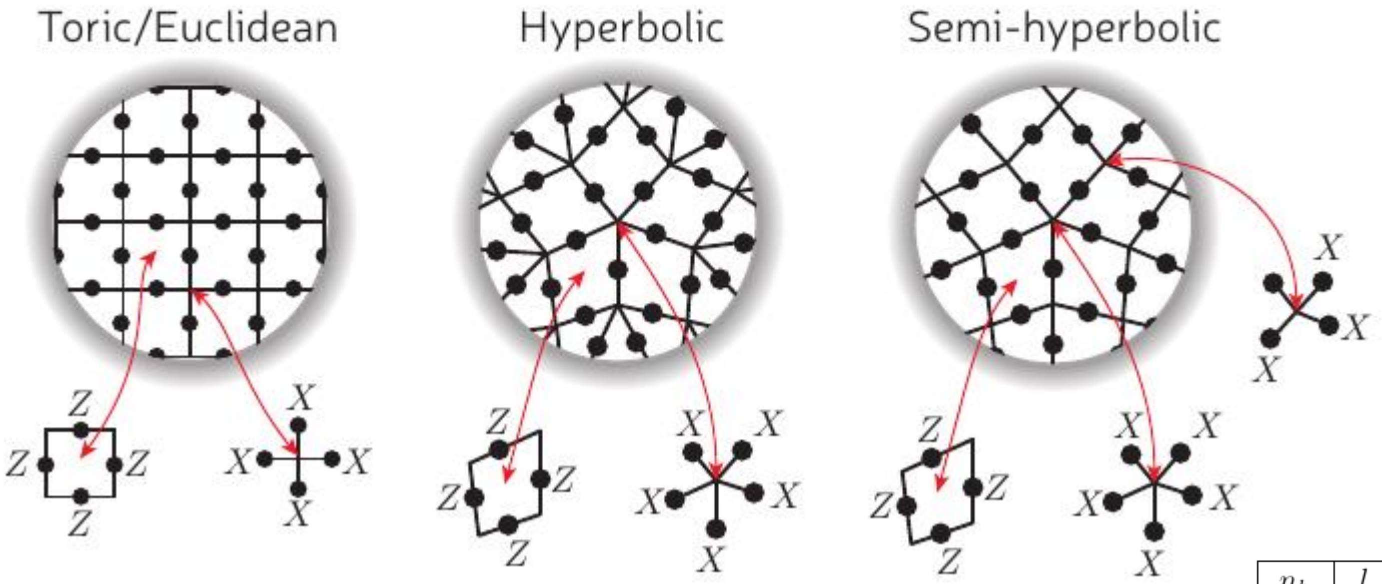


Fig. from S. Benjamin & J. Kelly, *Superconducting Qubits: Solving a wonderful problem*.
News & Views, Nature Materials 14, 561–563 (2015)

Hyperbolic Surface Codes



{4,5} tiling and interpolations between {4,5} tiling and {4,4} tiling (toric code). Asymptotic rate of {4,5} is k/n → 1/10 (without ancilla qubits) with *growing distance*

Hyperbolic code [[1800, 182, 10]] compare with 182 surface codes with [[100, 1,L=10]]: 18200 qubits!

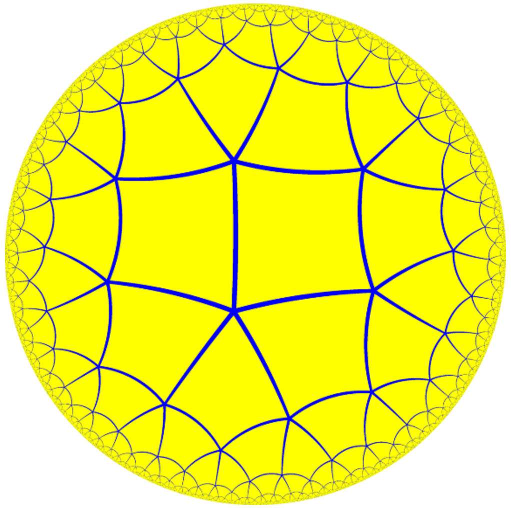
n_h	l	n	k	$d(\overline{Z})$	$d(\overline{X})$
60	1	60	8	4	6
60	2	240	8	8	10
60	3	540	8	12	14
60	4	960	8	16	18
60	5	1500	8	20	22
60	10	6000	8	40	42
160	1	160	18	6	8
160	2	640	18	12	14
160	3	1440	18	18	20
160	4	2560	18	24	26
160	5	4000	18	30	32

n_h	l	n	k	$d(\overline{Z})$	$d(\overline{X})$
360	1	360	38	8	8
360	2	1440	38	16	16
360	3	3240	38	24	24
360	4	5760	38	32	32
360	5	9000	38	40	40
1800	1	1800	182	10	10

[1] Breuckmann, Terhal, IEEE Trans. Inf. Theory and ArXiv (2015)
[2] Breuckmann, Campbell, Krishna, Terhal, Vuillot, to appear

Use of block codes [[n,k,d]]: useful for protecting information on channels and in storage

Hyperbolic Surface Codes



Bravyi-Terhal-Poulin: $kd^2 = O(n)$
for euclidean surfaces

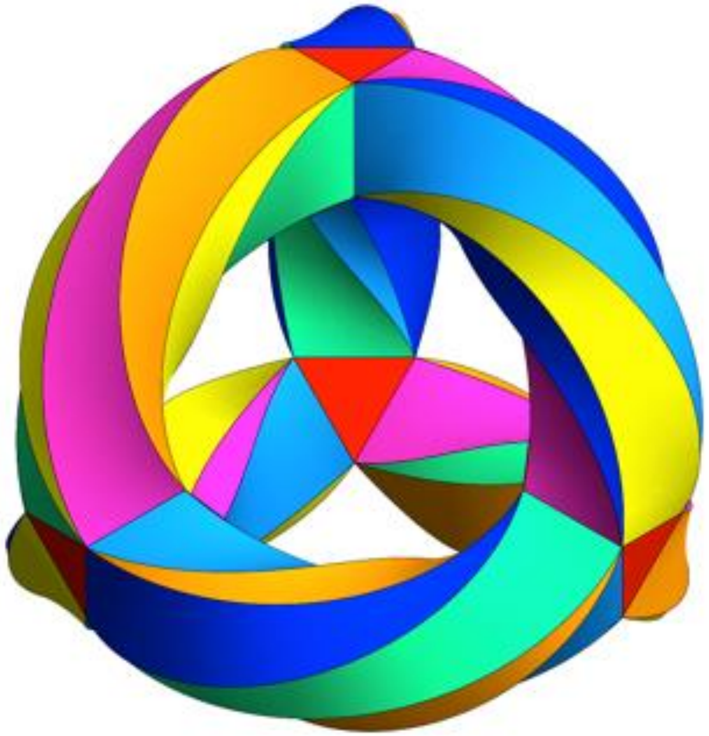
Delfosse: $kd^2 = O((\log k)^2 n)$ for any surface including hyperbolic ones.

Multi-handled torus where non-trivial loops determine what the logical operators are.

But placement of the qubits (tiling of surface) determines the distance of the code (logarithmically increasing with n for constant rate k/n).

Klein Quartic

http://www.gregegan.net/SCIENCE/KleinQuartic/KleinQuartic.html



{3,7}-tiling

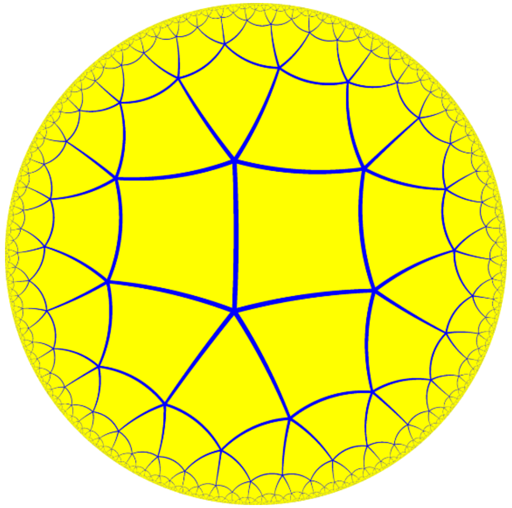
[[84,6,4]] code:
6 logical qubits (genus 3= max. number of handles one can cut without disconnect)
Distance 4 (logical Z weight 4, logical X weight 8)
Weight-3 Z-checks (triangular faces)
Weight-7 X-checks (vertices)

Bigger is possible, but this one has a lot of symmetries.

Home-made model
by Kasper Duivenvoorden
at RWTH Aachen

Model by Helaman Ferguson in marble and serpentine in Berkeley

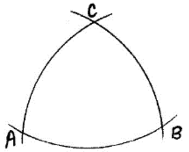
Creating a code



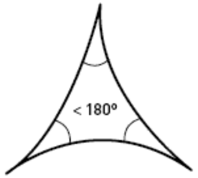
Regular {r,s}-tiling: r-gons with s of them meeting at a vertex.

{4,4},{6,3}, {3,6} only choices for a plane $\frac{1}{r}+\frac{1}{s}=\frac{1}{2}$

$\frac{1}{r}+\frac{1}{s}>\frac{1}{2}$ gives cube {4,3}, tetrahedron {3,3}, etc (Platonic solids)



$\frac{1}{r}+\frac{1}{s}<\frac{1}{2}$ gives tilings of hyperbolic plane



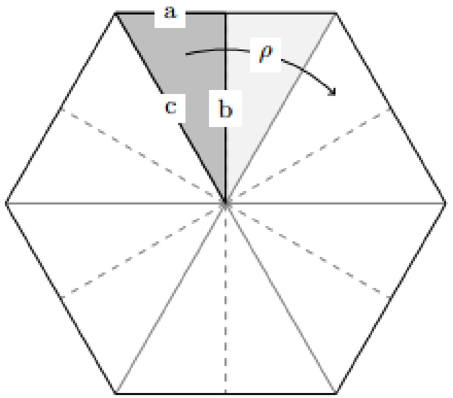
But one needs to wrap or close the surface up or identify r-gons.

Find a (normal, torsion-free) subgroup of translations under which triangles which make r-gons are identified (software) -> different subgroups for fixed {r,s} gives codes with different [[n,k,d]].

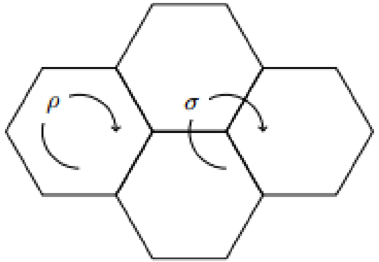
$$\frac{k}{n}=1-\frac{2}{r}-\frac{2}{s}+\frac{2}{n}$$

And distance $d \geq c \log n$

Distance computation is efficient.



(a) Action of the symmetry group on a single face.



(b) Rotations acting on the lattice

Fig. 6: Group acting on the $\{6,3\}$-tiling.

Conclusion

- Quantum error correction research is important as it is crucial in engineering a robust quantum computing device.

- Quantum error correction research is fun as it involves interesting math, fundamental physics and there are many open questions.

References:

- B.M. Terhal, *Quantum Error Correction for Quantum Memories*, Rev. Mod. Phys. 87, 307 (2015)

- E. Campbell, B.M. Terhal and C. Vuillot, *The Steep Road Towards Robust and Universal Quantum Computation*, arxiv.org: 1612.07330
See also 3D color code movie (made by C. Vuillot), https://youtu.be/erkeCxQ0-g4